

# Cluster structures in light nuclei

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**Abstract.** Clustering in light stable and unstable nuclei is discussed. After a brief review of the clustering in stable nuclei, we make a new prediction of the existence of the alpha cluster condensed states in  $^{12}\text{C}$  and  $^{16}\text{O}$ . Discussions of clustering in light unstable nuclei are made in the cases of Be and B isotopes up to the neutron dripline.

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## 1 Introduction

Due to the saturation property of nuclear binding energy, nucleons and nuclei easily assemble and disassemble without any large input or output of energy. The formation of clusters is a fundamental aspect of nuclear many-body dynamics, together with the formation of mean field [1]. In light stable nuclei, it is well known that the clustering structure is of basic importance [2]. We will show that the clustering is also of basic importance in light unstable nuclei. In the discussion of clustering in stable nuclei, I will make a new prediction of the existence of the alpha cluster condensed states. For the problem of clustering in neutron-rich nuclei, I will stress novel features of clustering which appear to violate the basic requirements for the clustering in stable nuclei. We will make a conjecture that the novel type of clustering may be in general an important candidate for nuclear structure near the neutron dripline [1].

## 2 Clustering in stable nuclei

As typical evidence of clustering in stable nuclei, we consider the following four items: 1) large width of cluster decay, 2) inversion doublet, 3) unification of bound, resonance, and scattering states, and 4) weak coupling between inter-cluster relative motion and internal excitation of cluster.

The first two items are best seen in  $^{16}\text{O}$ , where we have a  $K^\pi = 0^+$  rotational band upon the 6.06 MeV  $0^+$  state and a  $K^\pi = 0^-$  band upon the 9.63 MeV  $1^-$  state, and also in  $^{20}\text{Ne}$ , where we have the  $K^\pi = 0^+$  ground band and  $K^\pi = 0^-$  band upon the 5.78 MeV  $1^-$  state. These negative-parity rotational bands are located slightly above the  $\alpha$  decay threshold energies, 7.16 MeV in  $^{16}\text{O}$  and

4.73 MeV in  $^{20}\text{Ne}$ . In both nuclei, the negative-parity band members all have large  $\alpha$  decay widths with the magnitude of Wigner limit, and the positive-parity band members are all populated with large yields by  $\alpha$  transfer reactions. Furthermore, in both nuclei, the splitting energy ( $\Delta E = E(1^-) - E(0^+)$ ) between band head levels of the two bands is much less than the value of  $\hbar\omega$  ( $\approx 15$  MeV), and the moments of inertia of the positive- and negative-parity bands are almost equal to each other. These facts imply that the two rotational bands with  $K^\pi = 0^\pm$  in both  $^{16}\text{O}$  and  $^{20}\text{Ne}$  constitute inversion doublets and have the  $\alpha + ^{12}\text{C}$  and  $\alpha + ^{16}\text{O}$  cluster structures, respectively [2].

If the spatially localized clusters have reality, the relative motion between clusters should explain not only bound and resonance states but also scattering states. This implies that the cluster model for bound and resonance states should be unified with the optical potential model of cluster scattering. It is now known that this unification has been achieved in the case of the  $\alpha$  cluster [3] and is also almost true in the cases of some heavier clusters [3, 4]. In the case of the  $\alpha + ^{16}\text{O}$  system, the microscopic cluster model can reproduce not only the inversion doublet band states with  $K^\pi = 0^\pm$ , but also the differential cross-sections up to about 150 MeV, which exhibit the so-called ALAS (anomalous large-angle scattering) phenomena below about 50 MeV and the so-called nuclear rainbow phenomena at higher energies around 100 MeV [3]. In describing the scattering states by the microscopic cluster model, the same imaginary potential as that used by the phenomenological optical model analysis was used [5].

In stable nuclei, the following two conditions have been considered to be basic for the cluster structure. The first condition is that the clusters should be stiff, namely, the clusters should not be easily excited. If the clusters are soft, they will not be able to keep their identity due to inter-cluster interaction and they will melt into the

mean-field structure. The second condition is the so-called threshold rule [2], which states that when a cluster state is formed, it will appear near the threshold energy. If a cluster state appears far below the threshold energy, the clusters will interact strongly and then dissolve into the mean-field structure.

One of the fundamental questions associated with nuclear clustering is what kind of cluster states can be expected to exist around the threshold energy of  $n\alpha$  breakup in self-conjugate  $4n$  nuclei. One possible answer to this question is the cluster state with  $n\alpha$  linear chain structure. The idea of the  $\alpha$  linear chain state, which is originally put forth by Morinaga [6], is so fascinating that recently the existence of  $6\alpha$  linear chain states in  $^{24}\text{Mg}$  was studied extensively by experiments and theoretical analyses of them [7]. The possibility of the  $3\alpha$  linear chain state in  $^{12}\text{C}$ , which is the simplest  $\alpha$  linear chain state, was studied in detail by many authors by solving the  $3\alpha$  problem microscopically [8]. These three-body studies all showed that the  $3\alpha$  cluster states around the  $3\alpha$  threshold energy do not have the linear chain structure. The calculated second  $0^+$  state, which corresponds to the observed second  $0^+$  state located 0.39 MeV above the  $3\alpha$  threshold energy, was found to have the structure where  $\alpha$  clusters interact with each other dominantly in the relative  $S$ -wave. Thus, the theory concluded that the cluster state near the  $3\alpha$  threshold energy has not the  $3\alpha$  linear chain structure but an  $\alpha$ -gas-like structure that can be approximately expressed by the wave function

$$\mathcal{A}\{e^{-\gamma(\mathbf{X}_1^2+\mathbf{X}_2^2+\mathbf{X}_3^2)}\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}, \quad (1)$$

where  $\mathbf{X}_i$  stands for the center-of-mass coordinate of the  $i$ -th  $\alpha$  cluster  $\alpha_i$  and  $\phi(\alpha_i)$ , the internal wave function of the  $\alpha$  cluster  $\alpha_i$ . It is to be noted that this wave function of eq. (1) expresses the state where three  $\alpha$  clusters occupy the same  $0s$  harmonic oscillator orbit  $\exp[-\gamma\mathbf{X}^2]$ . Namely, the wave function of eq. (1) expresses a  $3\alpha$  cluster condensed state. Recently, the possibility of a  $4\alpha$  cluster condensed state was investigated by adopting a similar wave function to eq. (1) [9]. It was shown that we can expect the existence of the  $4\alpha$  cluster condensed state near the  $4\alpha$  threshold energy.

### 3 Clustering in neutron-rich nuclei

We discussed the clustering in neutron-rich nuclei on the basis of the results obtained by AMD (antisymmetrized molecular dynamics). The AMD studies of Be isotopes [10, 11] present a very good example that shows the importance of clustering in neutron-rich nuclei. AMD calculations show the existence of the  $\alpha$ - $\alpha$  core in all Be isotopes from  $^8\text{Be}$  up to the neutron dripline nucleus  $^{14}\text{Be}$ . We explained that the motion of neutrons is well understood by the concept of the molecular orbits around the  $\alpha$ - $\alpha$  core. A long time ago, Seya, Kohno, and Nagata extended the structure study by modeling molecular orbit up to the neutron dripline in Be and B isotopes [12]. The *ab initio*

model AMD now confirmed theoretically the formation of the  $\alpha$ - $\alpha$  core in Be isotopes up to the neutron dripline.

One important valence orbit for neutrons is the so-called  $\sigma$  orbit coming down from the  $sd$ -shell due to the clustering deformation. Particularly in  $^{10}\text{Be}$ , we could classify the observed states into four rotational bands:  $K^\pi = 0_2^+$  and  $K^\pi = 0_1^-$  bands were interpreted to have the neutron configurations with two neutrons and one neutron in the  $\sigma$  orbit, respectively while  $K^\pi = 0_1^+$  and  $K^\pi = 2_1^+$  bands were interpreted to have neutron configurations with no neutrons in  $\sigma$  orbit. We showed that the Gamow-Teller transition strengths from the  $^{10}\text{B}$  ground state to  $K^\pi = 0_1^+$  and  $K^\pi = 2_1^+$  band states are well reproduced by AMD [11].

If we adjust the parameters of the effective nuclear force so that the ground state of  $^{11}\text{Be}$  has the structure with one valence neutron occupying the  $\sigma$  orbit and has the spin parity  $1/2^+$ , the ground state of  $^{12}\text{Be}$  will have not the neutron-closed-shell structure but the structure with two valence neutrons occupying the  $\sigma$  orbit. This neutron-core-excited structure, which is in accordance with the recent experiments [13], gives us a better reproduction of the beta decay rate of the ground state to the ground state of  $^{11}\text{B}$  than the neutron-closed-shell structure [14].

We also reported the AMD study of B isotopes [15]. According to AMD, B isotopes near the neutron dripline nucleus  $^{19}\text{B}$  were shown to have prominent di-cluster density distribution. Here, five protons are divided spatially into two groups with two and three protons, respectively, which are surrounded by neutrons. The reliability of such AMD results was ensured by the good reproduction of data for  $^{13}\text{B} \sim ^{19}\text{B}$  which include binding energies, radii, electric quadrupole moments, and magnetic moments. We note an important fact in that the clustering in  $^{17}\text{B}$  and  $^{19}\text{B}$  is entirely different from the clustering feature in stable nuclei. For example, if we regard  $^{19}\text{B}$  as having a di-cluster structure of  $^8\text{He} + ^{11}\text{Li}$  or  $^{10}\text{He} + ^9\text{Li}$ , we observe that any of the clusters  $^8\text{He}$ ,  $^{10}\text{He}$ ,  $^9\text{Li}$ , and  $^{11}\text{Li}$  will not satisfy the ‘‘cluster condition’’ that the cluster should be a stable nucleus. Furthermore, we notice that the threshold rule [2] is heavily violated, because  $^{19}\text{B}$  is deeply bound by 12.73 MeV and 14.17 MeV from the  $^8\text{He} + ^{11}\text{Li}$  and  $^{10}\text{He} + ^9\text{Li}$  thresholds, respectively. (Also the neutron dripline nucleus  $^{14}\text{Be}$  is deeply bound by 9.16 MeV and 11.13 MeV from the  $^8\text{He} + ^6\text{He}$  and  $^{10}\text{He} + \alpha$  thresholds, respectively.) Thus we are forced to conclude that the clustering in  $^{17}\text{B}$  and  $^{19}\text{B}$  is of a novel type, different from the clustering in stable nuclei.

A conjecture was made that in general, a novel type of clustering may be an important candidate for the nuclear structure near the neutron dripline [1]. The argument for this conjecture is as follows. By definition, the neutron dripline nucleus has a structure that accommodates the maximum number of neutrons for a given number of protons. Such a structure is realized by making the volume of the neutron skin region maximum by avoiding the energy loss due to the Pauli principle and shell effects. Since the thickness of the neutron skin cannot be larger than the nuclear-force range on account of the fact that

neutrons without protons cannot be bound by themselves, we see that the deformed shape has larger thickness of neutron skin than the spherical shape and that the clustering shape with spatially divided protons has an even larger thickness of neutron skin.

## 4 Conclusion

Clustering is of fundamental importance in nuclear many-body dynamics and shows up in a variety of phenomena. In the excitation energy region near the  $n\alpha$  break-up threshold in  $N = Z = 4n$  nuclei, the alpha-cluster condensed state may show up as a new type of clustering state. In neutron-rich nuclei, clustering is expected to play an even more important role than in stable nuclei, taking novel types of appearance.

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